

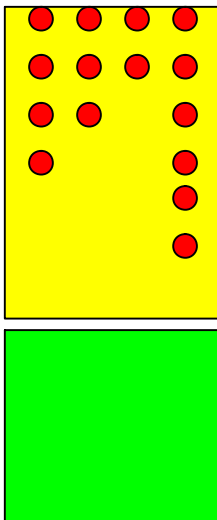
# Playing with integer numbers

We suggest a game on algebraic operations with integer numbers. The material we are going to need is a two-coloured baize (i.e. yellow-green) and two coloured small balls (i.e. red and white). Initially, we have to agree on some basic rules:

- Red balls stand for positive integers and White balls stand for negative integers.
- When we add integers, we place the correspondent balls on the yellow baize, while when we subtract integers we place the correspondent balls on the green baize.
- When we add, every white ball is extinguished by a red ball. This is achieved through a simple movement of a player to withdraw the two balls from the baize.
- If we move balls from one part of the baize i.e. the yellow one to the other i.e. the green one, the balls change colour. This “transformation” technique is useful for the subtraction and the multiplication of the integers.

Now, let's play!

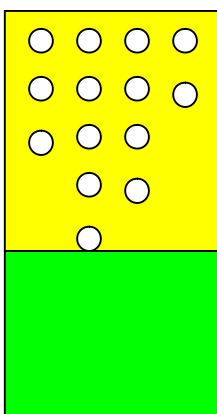
## 1<sup>st</sup> example: Addition of positive integers



$$(+4)+(+3)+(+2)+(+6)=(+17)$$

Therefore if we add 4 and 3 and 2 and 6 red balls we'll have 17 red balls

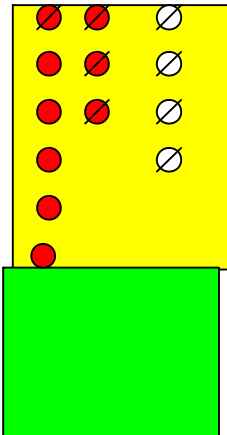
## 2<sup>nd</sup> example: Addition of negative integers



$$(-3)+(-5)+(-4)+(-2)=(-14)$$

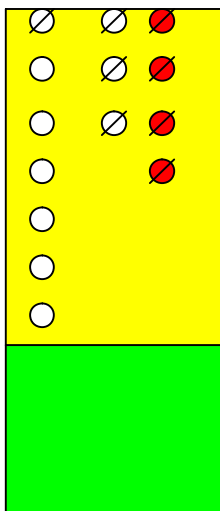
if we add 3 and 5 and 4 and 2 white balls we'll have 14 white balls.

3<sup>rd</sup> example: Addition of positive and negative integers



a)  $(+6)+(+3)+(-4)=(+5)$

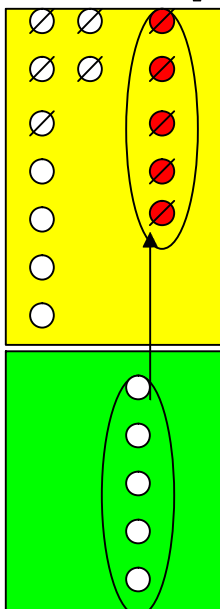
if we add 6 red balls and 3 red balls and 4 white balls that will be canceled out by 4 red balls, we'll have 5 red balls.



b)  $(-7)+(-3)+(+4)=(-6)$

if we add 7 white balls and 3 white balls and 4 red balls that will be canceled out by 4 white balls, we'll find 6 white balls.

4<sup>th</sup> example: Addition and subtraction of positive and negative integers



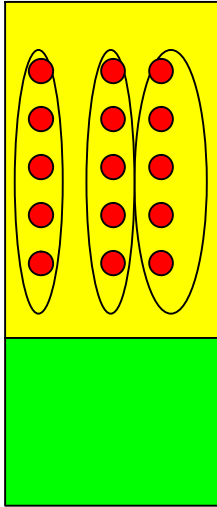
$(-7)+(-2)-(-5)=(-7)+(-2)+(+5)=(-4)$

In this example, the 5 white balls which are being subtracted by the yellow baize. There they will be added to 9 white balls. Therefore, the 5 red balls cancel out 5 white balls so finally we'll have 4 white balls.

## Multiplication of integers

### 1<sup>st</sup> example: Multiplication of positive integers

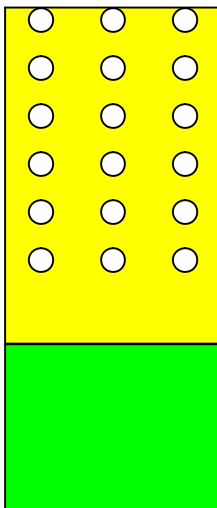
Multiplication of integers has really to do with the operation of addition. For instance,  $4 \times 5 = 5 + 5 + 5 + 5$ . Our game is based on this relationship.



We play on the yellow baize. We add 5 and 5 and 5 red balls which means 3 pentads of red balls. So we'll have finally 15 red balls. Therefore:

$$(+3).(+5)=(+5)+(+5)+(+5)=(+15)$$

### 2<sup>nd</sup> example: Multiplication of a positive integer with a negative integer



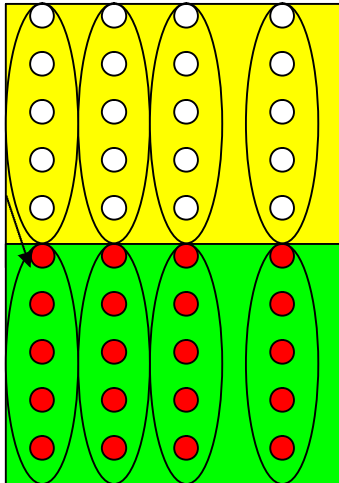
Similarly, on the yellow baize we add 6 and 6 and 6 white balls which means 3 hexads of white balls. So finally we'll have 18 balls

$$(+3).(-6)=(-6)+(-6)+(-6)=(-18)$$

### 3<sup>rd</sup> example: Multiplication of negative integers

This operation is achieved by using the “transformation technique”. We put on the yellow baize 4 pentads of white balls. On the green baize we add 4 transformed pentads, which means that we add 4 pentads of red balls so finally we’ll have 20 red balls. Therefore:

$$(-4).(-5)$$



$$(-4).(-5)=(+5)+(+5)+(+5)+(+5)=(+20)$$

↓   ↓   ↓   ↓  
1<sup>st</sup> 2<sup>nd</sup> 3<sup>rd</sup> 4<sup>th</sup>  
transformed pentad

**NOW IT'S YOUR TURN! HAVE FUN WITH THE GAME!**

**PLAY AND FIND THE FOLLOWING:**

1.  $(+3).(+8)$ ,  $(-2).(+6)$ ,  $(-3).(-7)$
2.  $(-6)+(-3)+(-5)+(+5)$
3.  $(-8)-(-3)+(-5)-(-2)$